

# Solid quantization for non-point particles

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In quantum field theory, elemental particles are assumed to be point particles. As a result, the loop integrals are divergent in many cases. Regularization and renormalization are necessary in order to get the physical finite results from the infinite, divergent loop integrations. We propose new quantization conditions for non-point particles. With this solid quantization, divergence could be treated systematically. This method is useful for effective field theory which is on hadron degrees of freedom. The elemental particles could also be non-point ones. They can be studied in this approach as well.

Quantum field theory is the fundamental theory for nuclear and particle physics. The simplest way to quantize the field is to use canonical quantization which is similar as in quantum mechanics. It is equivalent to the path integral method. With the quantum field theory, one can study the micro process with Feynman rules. When do the high order calculation, the loop contribution will appear. These integrals are often divergent, i.e., they become infinite when momentum integration goes to infinity. This ultraviolet divergence is short-distance phenomenon.

Many kinds of methods are introduced in quantum field theory to deal with the divergence. One of the most popular method is dimensional regularization [1]. It provides a systematic tool to obtain finite physical results from the infinity. Another is Pauli-Villars regularization which adds fictitious particles to the theory with large masses to cancel out the infinity [2].

Quantum field theory with dimensional regularization is very standard and widely accepted. It is also applied in effective field theory which is on hadron degrees of freedom [3–5]. In hadron physics, there are a lot of phenomenological models where divergence is often treated by adding a cutoff or form factor to the integral “by hand”. The cutoff or form factor can be related to the wave function which means hadrons are not point particles [6, 7]. It can also be “derived” from the non-local interaction [8, 9]. In other words, if particles are not point ones, there is no divergence appear from the beginning. There exists quantum field theories for point particles. Whether we can have some “theories” for non-point particles in which divergence can be avoided systematically? We will show that from the new quantization conditions, one could get the modified propagators for non-point particles. Divergence can be treated systematically.

In fact, in the early 1950s, Yukawa has proposed the non-local fields which described the non-point particles [10, 11]. It was assumed that the non-local field was a function of four space-time operators  $x_\mu$  as well as of four space-time displacement operators  $p_\mu$ . Besides the normal equation, this field satisfied another one which was related to the radius of elemental particle. However, this

idea did not get widely accepted because the aim to get rid of divergence was not easily established [12, 13]. In Refs. [14, 15], the authors claimed that there exists no meaningful  $S$  matrix with non-local interaction. While some authors pointed out that the violation of unitarity observed in space/time noncommutative field theories was due to an improper definition of quantum field theory on noncommutative spacetime (Quantum field theory on the standard noncommutative spacetime is equivalent to a non-local theory on a commutative spacetime.) [16]. As long as a proper perturbative setup is employed, non-local field theories may well be unitary in the sense that probabilities are always conserved. A proof of unitarity of  $S$  matrix as well as causality in a non-local quantum field theory has been shown in the paper of Alebastrov and Efimov [17, 18]. At the same time, non-local quantum electrodynamics was widely discussed [19–24]. In recent years, a lot of work has been done on the non-local phenomenological models as well as on the noncommutative field theory [25–30]. For practise, one can use the unitary operator  $T \exp\{i \int_{-\infty}^{\infty} d^4x \mathcal{L}_{int}(x)\}$ , where  $\mathcal{L}_{int}(x)$  is the non-local interaction, to do the perturbative expansion order by order [8, 31].

In this paper, we propose new quantization conditions for non-point particles. Consistent with this solid quantization, the non-local Lagrangian is straightforward. Different from the traditional non-local case, here the free Lagrangian should be non-local as well.

Let's start with the traditional canonical quantization for the simplest scalar field. The traditional commutation relations are:

$$\begin{aligned} [\phi(\vec{x}, t), \phi(\vec{y}, t)] &= [\pi(\vec{x}, t), \pi(\vec{y}, t)] = 0, \\ [\phi(\vec{x}, t), \pi(\vec{y}, t)] &= i\delta^{(3)}(\vec{x} - \vec{y}). \end{aligned} \quad (1)$$

The  $\delta$  function in the above equation means that a point particle and anti-particle can only be created at the same position point.

The field and its conjugate partner can be expanded in momentum space, expressed as

$$\phi(\vec{x}, t) = \int \widetilde{d\vec{p}} [a(\vec{p})e^{i\vec{p}\cdot\vec{x} - i\omega_p t} + a^\dagger(\vec{p})e^{-i\vec{p}\cdot\vec{x} + i\omega_p t}], \quad (2)$$

$$\pi(\vec{x}, t) = \int \widetilde{dp} (-i)\omega_p [a(\vec{p})e^{i\vec{p}\cdot\vec{x}-i\omega_p t} - a^\dagger(\vec{p})e^{-i\vec{p}\cdot\vec{x}+i\omega_p t}], \quad (3)$$

where

$$\widetilde{dp} = \frac{d^3p}{(2\pi)^3 2\omega_p}. \quad (4)$$

It is straightforward to obtain the commutation relations between creation and annihilation operators:

$$\begin{aligned} [a(\vec{p}), a(\vec{q})] &= [a^\dagger(\vec{p}), a^\dagger(\vec{q})] = 0, \\ [a(\vec{p}), a^\dagger(\vec{q})] &= (2\pi)^3 2\omega_p \delta^{(3)}(\vec{p} - \vec{q}). \end{aligned} \quad (5)$$

The creation operator creates a momentum state  $|p\rangle = a^\dagger(\vec{p})|0\rangle$  which is normalized as

$$\int \widetilde{dp} |p\rangle \langle p| = 1. \quad (6)$$

Because the particle is assumed to be point particle (behaves like  $\delta$  function in position space), when expanded in momentum space, it has the same possibility for different momentum. However, the real particle could be like a wavepacket. It is partially localized in both position and momentum space. The possibility of the particle with high momentum is small. With high-momentum suppression, the divergence in the loop integral may not appear.

Therefore, we propose new quantization conditions (solid quantization):

$$\begin{aligned} [\phi(\vec{x}, t), \phi(\vec{y}, t)] &= [\pi(\vec{x}, t), \pi(\vec{y}, t)] = 0, \\ [\phi(\vec{x}, t), \pi(\vec{y}, t)] &= i\Phi(\vec{x} - \vec{y}). \end{aligned} \quad (7)$$

The function  $\Phi(\vec{x} - \vec{y})$  describes the correlation between fields at  $\vec{x}$  and  $\vec{y}$ . Due to the fact that particle is not a dimensionless point particle, but a solid one, particles at different positions could be partially superimposed which means there exists some possibility that particle and antiparticle are created in different positions.

One can also expand the field as Eq. (2) (In this case, we use capital letter  $A$  instead of  $a$ .)

$$\phi(\vec{x}, t) = \int \widetilde{dp} [A(\vec{p})e^{i\vec{p}\cdot\vec{x}-i\omega_p t} + A^\dagger(\vec{p})e^{-i\vec{p}\cdot\vec{x}+i\omega_p t}]. \quad (8)$$

As a result, the creation and annihilation operators satisfy the following relations

$$\begin{aligned} [A(\vec{p}), A(\vec{q})] &= [A^\dagger(\vec{p}), A^\dagger(\vec{q})] = 0, \\ [A(\vec{p}), A^\dagger(\vec{q})] &= (2\pi)^3 2\omega_p \delta^{(3)}(\vec{p} - \vec{q}) \Psi(\vec{p}). \end{aligned} \quad (9)$$

$\Phi(\vec{x})$  and  $\Psi(\vec{p})$  obey the following relations

$$\Phi(\vec{x}) = \int \frac{d^3p}{(2\pi)^3} \frac{\Psi(\vec{p})}{2} (e^{i\vec{p}\cdot\vec{x}} + e^{-i\vec{p}\cdot\vec{x}}), \quad (10)$$

$$\Psi(\vec{p}) = \int d^3x \frac{\Phi(\vec{x})}{2} (e^{i\vec{p}\cdot\vec{x}} + e^{-i\vec{p}\cdot\vec{x}}). \quad (11)$$

The above two equations generate two normalization formulas

$$\Phi(0) = \int \frac{d^3p}{(2\pi)^3} \Psi(\vec{p}), \quad (12)$$

$$\Psi(0) = \int d^3x \Phi(\vec{x}) = 1. \quad (13)$$

Compared with the traditional commutation relation where  $\Phi(\vec{x}) = \delta^{(3)}(\vec{x})$ ,  $\Phi(\vec{x})$  is normalized to be 1, while  $\Psi(\vec{p})$  is normalized to be  $\Phi(0)$ .

With the new quantization, the field can be written in terms of traditional creation and annihilation operators as

$$\phi(\vec{x}, t) = \int \widetilde{dp} \sqrt{\Psi(\vec{p})} [a(\vec{p})e^{i\vec{p}\cdot\vec{x}-i\omega_p t} + a^\dagger(\vec{p})e^{-i\vec{p}\cdot\vec{x}+i\omega_p t}]. \quad (14)$$

It is easy to get the Feynman propagator of the scalar field in the solid quantization. The propagator is defined as

$$\begin{aligned} \Delta_F(x' - x) &= \langle 0 | T \phi(x') \phi(x) | 0 \rangle \\ &= \int \widetilde{dk} [\theta(t' - t) e^{ik \cdot (x' - x)} + \theta(t - t') e^{-ik \cdot (x' - x)}]. \end{aligned} \quad (15)$$

The integral expression of the step function is

$$\theta(t) = \lim_{\epsilon \rightarrow 0^+} \int \frac{d\tau}{2\pi i} \frac{e^{i\tau t}}{\tau - i\epsilon}. \quad (16)$$

With the help of the above equation, the Feynman propagator can be obtained as

$$\Delta_F(x' - x) = \int \frac{d^4k}{(2\pi)^4} \frac{i\Psi(\vec{k}) e^{-ik \cdot (x' - x)}}{k^2 - m^2 + i\epsilon}. \quad (17)$$

For the other fields, the quantization condition is similar. For example, for spin 1/2 fermion, the nonzero anti-commutation relationship is

$$\{\psi_\alpha(\vec{x}, t), \bar{\psi}_\beta(\vec{y}, t)\} = \gamma_{\alpha\beta}^0 \Phi(\vec{x} - \vec{y}). \quad (18)$$

Correspondingly, the field should be written as

$$\begin{aligned} \psi(\vec{x}, t) &= \sum_{s=\pm} \int \widetilde{dp} \sqrt{\Psi(\vec{p})} [b_s(\vec{p}) u_s(\vec{p}) e^{i\vec{p}\cdot\vec{x}-i\omega_p t} \\ &\quad + d_s^\dagger(\vec{p}) v_s(\vec{p}) e^{-i\vec{p}\cdot\vec{x}+i\omega_p t}], \end{aligned} \quad (19)$$

where  $b$  and  $d^\dagger$  are normal annihilation and creation operators.  $u_s(\vec{p})$  and  $v_s(\vec{p})$  are Dirac spinors. The propagator of the spin 1/2 field can be obtained as

$$S_F(x' - x) = \int \frac{d^4k}{(2\pi)^4} \frac{i\Psi(\vec{k})(k \cdot \gamma + m) e^{-ik \cdot (x' - x)}}{k^2 - m^2 + i\epsilon}. \quad (20)$$

Vector field, say photon field can also be expanded as

$$A^\mu(\vec{x}, t) = \sum_{\lambda=\pm} \int \widetilde{d\vec{p}} \sqrt{\Psi(\vec{p})} [a_\lambda(\vec{p}) \epsilon^\mu(\vec{p}, \lambda) e^{i\vec{p}\cdot\vec{x} - i\omega_p t} + a_\lambda^\dagger(\vec{p}) \epsilon^{*\mu}(\vec{p}, \lambda) e^{-i\vec{p}\cdot\vec{x} + i\omega_p t}], \quad (21)$$

where  $\epsilon^\mu(\vec{p}, \lambda)$  is the polarization vector. The photon propagator can be written as

$$D_F^{\mu\nu}(x' - x) = \int \frac{d^4 k}{(2\pi)^4} \frac{-i\Psi(\vec{k}) g^{\mu\nu} e^{-ik\cdot(x' - x)}}{k^2 - m^2 + i\epsilon}. \quad (22)$$

We should mention that, in principle, the function  $\Psi(\vec{p})$  or  $\Phi(\vec{x} - \vec{y})$  is particle dependent. It describes the particle's property in addition to the mass and width. Therefore, with the new quantization conditions, the Feynman rules should be changed correspondingly. The new propagator of the field is multiplied by a factor  $\Psi(\vec{k})$  and the external field is multiplied by a factor  $\sqrt{\Psi(\vec{k})}$ .

A question may arise here that how to connect the new propagator with the path integral formulation. The path integral for the free point-like field is defined as

$$Z_0(J) = \int \mathcal{D}\phi e^{i \int d^4 x [\mathcal{L}_0 + J\phi]}, \quad (23)$$

where

$$\mathcal{L}_0 = -\frac{1}{2} \partial^\mu \phi \partial_\mu \phi - \frac{1}{2} m^2 \phi^2 \quad (24)$$

is the Lagrangian density and  $J$  is the external current. For a solid particle, the free Lagrangian density is different. The density can be written as

$$\mathcal{L}_0 = \phi \frac{(\partial^\mu \partial_\mu - m^2)}{2\Psi(i\vec{\partial})} \phi. \quad (25)$$

With the above Lagrangian density, the propagator of scalar field obtained in the path integral formulation is the same as that in solid canonical quantization. For the fermion and vector fields, the situation is the same.

The factor  $\Phi(\vec{x} - \vec{y})$  is the correlation of two particles at  $\vec{x}$  and  $\vec{y}$ . If we choose  $\Phi(\vec{x} - \vec{y}) = \delta^{(3)}(\vec{x} - \vec{y})$ ,  $\Psi(\vec{p})$  will equal 1. All of the above propagators will be changed back to the conventional ones. As we explained previously, the particle could be a solid particle with three space dimensions. The particle and antiparticle can be created at small distance. Therefore, the function of  $\Phi(\vec{x} - \vec{y})$  can be a function which decreases with the increasing distance  $|\vec{x} - \vec{y}|$ . The smaller the particle, the closer the function to  $\delta$  function.

The above solid quantization provides the “kinematics” for quantum field theory. Now let's look at the “dynamics” for non-point particles. Gauge invariance is a fundamental method to get the strong or electro-weak interactions. Since particles are not point ones, in general, the interaction among them is non-local. Similar as

the non-local quark-meson interaction [8, 31], the gauge invariant interaction between fermion and gauge field, say photon field, can be written as

$$\mathcal{L}(x) = \int d^3 a \bar{\psi}(t, \vec{x} + \frac{\vec{a}}{2}) e^{iI(\vec{x} + \vec{a}/2, \vec{x})} \gamma^\mu i D_\mu e^{-iI(\vec{x} - \vec{a}/2, \vec{x})} \psi(t, \vec{x} - \frac{\vec{a}}{2}) F(\vec{a}), \quad (26)$$

where  $D_\mu = \partial_\mu - ig \int d^3 b A_\mu(t, \vec{x} + \vec{b}) G(\vec{a}, \vec{b})$  and  $I(y, x) = g \int_x^y dz_\mu \int d^3 b A^\mu(z_0, \vec{z} + \vec{b}) G(\vec{a}, \vec{b})$ .  $g$  is the coupling constant. The function  $F(\vec{a})$  and  $G(\vec{a}, \vec{b})$  are related to the size of fermion and gauge fields. The non-local coupling depends on the distance between the two fermion fields and the distance between gauge field and the center of two fermion fields. The coupling  $G(\vec{a}, \vec{b})$  can be factorized as  $\frac{\Phi(\vec{a}) \Phi_g(\vec{b})}{F(\vec{a})}$ , where  $\Phi(\vec{a})$  is the correlation function between two fermions at distance  $\vec{a}$  defined in Eq. (7).  $\Phi_g(\vec{b})$  the correlation function for gauge fields. The particular choice of  $G(\vec{a}, \vec{b})$  is to get the interacting term  $\int d^3 a \int d^3 b \bar{\psi}(t, \vec{x} + \frac{\vec{a}}{2}) \gamma^\mu \psi(t, \vec{x} - \frac{\vec{a}}{2}) A_\mu(t, \vec{x} + \vec{b}) \Phi(\vec{a}) \Phi_g(\vec{b})$  which provides the possibility  $\Phi(\vec{a}) \Phi_g(\vec{b})$  for the non-local interaction.

The above Lagrangian is invariant under the following gauge transformation:

$$\begin{aligned} \psi(t, \vec{x} - \vec{a}/2) &\rightarrow e^{ig_{\text{eff}}\theta(t, \vec{x} - \vec{a}/2)} \psi(t, \vec{x} - \vec{a}/2), \\ \bar{\psi}(t, \vec{x} + \vec{a}/2) &\rightarrow \bar{\psi}(t, \vec{x} + \vec{a}/2) e^{-ig_{\text{eff}}\theta(t, \vec{x} + \vec{a}/2)}, \\ A_\mu(t, \vec{x} + \vec{b}) &\rightarrow A_\mu(t, \vec{x} + \vec{b}) + \partial_\mu \theta'(t, \vec{x} + \vec{b}), \end{aligned} \quad (27)$$

where  $g_{\text{eff}}$  is defined as  $\frac{\Phi(\vec{a}) \Phi_g(\vec{b})}{F(\vec{a})}$  which can be understood as the effective charge of a non-local electromagnetic current with a distance  $\vec{a}$  between a fermion and an anti-fermion. The appearance of  $F(\vec{a})$  in the denominator of  $g_{\text{eff}}$  is because of the non-point property of the fermions. The functions  $\theta$  and  $\theta'$  have the following relation

$$\theta(t, \vec{x}) = \int d^3 b \theta'(t, \vec{x} + \vec{b}). \quad (28)$$

The strong and weak interaction can be easily obtained in the same way with  $SU(3)$  and  $SU(2)$  generators.

The free Lagrangian density without gauge field is

$$\mathcal{L}_0(x) = \int d^3 a \bar{\psi}(t, \vec{x} + \frac{\vec{a}}{2}) \gamma^\mu i \partial_\mu \psi(t, \vec{x} - \frac{\vec{a}}{2}) F(\vec{a}). \quad (29)$$

This non-local free Lagrangian is different from that in the traditional non-local models where the free part of the Lagrangian is local and the interaction part is a non-local coupling of point particles [8, 25–28, 31]. Our treatment is more consistent. Due to the non-point assumption, the non-local Lagrangian is straightforward and it is also necessary because of the solid quantization. After moving the position to the same point by the translation operator, it is straightforward to rewrite the above Lagrangian

as

$$\mathcal{L}_0(x) = \bar{\psi}(x) \gamma^\mu i \partial_\mu \tilde{F}(i\vec{\partial}) \psi(x), \quad (30)$$

where  $\tilde{F}(i\vec{\partial})$  is the Fourier transformation of  $F(\vec{a})$ , i.e.,

$$\tilde{F}(i\vec{\partial}) = \int d^3a e^{i\vec{a} \cdot i\vec{\partial}} F(\vec{a}). \quad (31)$$

Comparing Eqs. (25) and (30), we can get the relationship  $\tilde{F}(i\vec{\partial}) = 1/\Psi(i\vec{\partial})$ . One can see that the solid quantization is consistent with the path integral approach with non-local Lagrangian density.

The interaction term is written as

$$\begin{aligned} \mathcal{L}_{int}(x) = & g \int d^3a \int d^3b \bar{\psi}(t, \vec{x} + \frac{\vec{a}}{2}) e^{iI(\vec{x} + \vec{a}/2, \vec{x})} \gamma^\mu \\ & A_\mu(t, \vec{x} + \vec{b}) e^{-iI(\vec{x} - \vec{a}/2, \vec{x})} \psi(t, \vec{x} - \frac{\vec{a}}{2}) \Phi(\vec{a}) \Phi_g(\vec{b}) \\ & + \int d^3a \bar{\psi}(t, \vec{x} + \frac{\vec{a}}{2}) (e^{iI(\vec{x} + \vec{a}/2, \vec{x})} - 1) \gamma^\mu i \partial_\mu \\ & e^{-iI(\vec{x} - \vec{a}/2, \vec{x})} \psi(t, \vec{x} - \frac{\vec{a}}{2}) F(\vec{a}) \\ & + \int d^3a \bar{\psi}(t, \vec{x} + \frac{\vec{a}}{2}) \gamma^\mu i \partial_\mu (e^{-iI(\vec{x} - \vec{a}/2, \vec{x})} - 1) \\ & \psi(t, \vec{x} - \frac{\vec{a}}{2}) F(\vec{a}). \end{aligned} \quad (32)$$

The field can be expanded in power of  $\vec{a}$  and  $\vec{b}$  as

$$\begin{aligned} \psi(t, \vec{x} + \vec{a}) &= \psi(t, \vec{x}) + \vec{\partial} \psi(t, \vec{x}) \cdot \vec{a} + \mathcal{O}(\vec{a}^2), \\ A_\mu(t, \vec{x} + \vec{b}) &= A_\mu(t, \vec{x}) + \vec{\partial} A_\mu(t, \vec{x}) \cdot \vec{b} + \mathcal{O}(\vec{b}^2). \end{aligned} \quad (33)$$

The interaction can be expressed as

$$\mathcal{L}_{int}(x) = g \bar{\psi}(x) \gamma^\mu A_\mu(x) \psi(x) + \mathcal{O}(\vec{a}, \vec{b}), \quad (34)$$

where  $\vec{a}$  and  $\vec{b}$  reflect the size of the particles defined as

$$\begin{aligned} \vec{a} &= \int d^3a \vec{a} \Phi(\vec{a}), \\ \vec{b} &= \int d^3b \vec{b} \Phi_g(\vec{b}). \end{aligned} \quad (35)$$

For the “free” Lagrangian, we should not expand it in terms of  $\vec{a}$ . The “free” Lagrangian provides propagators for solid particles. The further volume effect of solid particles can be added order by order. If the particle’s size is small enough, we can neglect high order terms in Eq. (34). The lowest order interaction term is the same as that in the local case.

The solid quantization is valid for elemental particles as well as for hadrons if elemental particles are not point ones either. With the new propagator, the loop integration is convergent. For example, let’s look at the following integration which appears in the photon self-energy at one-loop level:

$$I = \int \frac{d^4k}{(2\pi)^4} \frac{\Psi(\vec{k}) \Psi(\vec{k} + \vec{p})}{[k^2 - m^2][(p + k)^2 - m^2]}, \quad (36)$$

where  $p$  is the external momentum of photon.  $k$  and  $k + p$  are the internal momentum of two electron or quark propagators. After integration of  $k_0$ , the above equation can be written as

$$I = \int \frac{d^3k}{2(2\pi)^3} \left\{ \frac{-i\Psi(\vec{k})\Psi(\vec{k} + \vec{p})}{\omega(\vec{k})[(\omega(\vec{k}) + \omega(\vec{p}))^2 - \omega^2(\vec{k} + \vec{p})]} - \frac{i\Psi(\vec{k})\Psi(\vec{k} + \vec{p})}{\omega(\vec{k} + \vec{p})[(\omega(\vec{k} + \vec{p}) - \omega(\vec{p}))^2 - \omega^2(\vec{k})]} \right\}, \quad (37)$$

where  $\omega(\vec{q}) = \sqrt{\vec{q}^2 + m^2}$ . Without the factor  $\Psi(\vec{k})$  and  $\Psi(\vec{k} + \vec{p})$ , the above integration is log-divergent. Since the particle is a solid one with three dimensions, its wavefunction is suppressed at high momentum. If we choose  $\Psi(\vec{k})$  to be a dipole or Gauss function, the integration is convergent.

Without renormalization, the running coupling constant can also be understood. With the new quantization conditions, even at tree level, the coupling constant will be associated with a momentum dependent factor. For example, for the fermion-boson coupling, if the initial and final momentum of fermions are  $-\vec{q}/2$  and  $\vec{q}/2$ , the momentum of the boson is  $\vec{q}$ , and the momentum dependent factor of the coupling constant at tree level is  $\sqrt{\Psi_f(-\vec{q}/2)\Psi_f(\vec{q}/2)\Psi_g(\vec{q})}$ . The labels  $f$  and  $g$  are for fermion and gauge boson, respectively. The asymptotic free is a general property not only for strong interaction. It is because the particle is not a point one. The momentum is partially localized which favors at low value.

Investigating quantum electrodynamic process is a good and clean way to test this quantization for elemental particles. Let’s study the electron-photon Compton scattering for an example:

$$e^-(p, s) + \gamma(k, \epsilon) \rightarrow e^-(p', s') + \gamma(k', \epsilon'), \quad (38)$$

where  $p$  and  $p'$ ,  $k$  and  $k'$  are the initial and final momentum of electron and photon, respectively.  $s$  and  $\epsilon$  are their spin and polarization. The scattering amplitude can be obtained as

$$\begin{aligned} \mathcal{M} = & -e^2 \sqrt{\Psi_e(\vec{p})\Psi_e(\vec{p}')\Psi_\gamma(\vec{k})\Psi_\gamma(\vec{k}')}\bar{u}_{s'}(\vec{p}') \left[ \Psi_e(\vec{p} + \vec{k}) \right. \\ & \left. \not{\epsilon}'^* \frac{1}{\not{p} + \not{k} - m} \not{\epsilon} + \Psi_e(\vec{p} - \vec{k}') \not{\epsilon} \frac{1}{\not{p} - \not{k}' - m} \not{\epsilon}'^* \right] u_s(\vec{p}). \end{aligned} \quad (39)$$

In the Lab frame where the initial electron is at rest, after summing over the initial and final electron spins, averaged square of amplitude is simplified as

$$\bar{\mathcal{M}}^2 = \frac{e^4}{8m^2} \left[ \frac{A}{\omega^2} + \frac{B}{\omega'^2} + \frac{C}{\omega\omega'} \right], \quad (40)$$

where  $\omega$  and  $\omega'$  are energy of initial and final photon.  $A$ ,  $B$  and  $C$  is expressed as

$$A = 8m\omega [2(k \cdot \epsilon')^2 + m\omega'] F_1(\omega, \omega'), \quad (41)$$

$$B = -8m\omega' [2(k' \cdot \epsilon)^2 - m\omega] F_2(\omega, \omega'), \quad (42)$$

$$C = [16m^2\omega\omega'[2(\epsilon \cdot \epsilon')^2 - 1] - 16m\omega'(k \cdot \epsilon')^2 + 16m\omega(k' \cdot \epsilon)^2] F_3(\omega, \omega'), \quad (43)$$

where

$$\begin{aligned} F_1(\omega, \omega') &= \Psi_e(\omega^2 + \omega'^2 - 2\omega\omega' \cos\theta) \Psi_\gamma(\omega^2) \\ &\quad \Psi_\gamma(\omega'^2) \Psi_e(\omega'^2), \\ F_2(\omega, \omega') &= \Psi_e(\omega^2 + \omega'^2 - 2\omega\omega' \cos\theta) \Psi_\gamma(\omega^2) \\ &\quad \Psi_\gamma(\omega'^2) \Psi_e(\omega'^2), \\ F_3(\omega, \omega') &= \Psi_e(\omega^2 + \omega'^2 - 2\omega\omega' \cos\theta) \Psi_\gamma(\omega^2) \\ &\quad \Psi_\gamma(\omega'^2) \Psi_e(\omega^2) \Psi_e(\omega'^2) \end{aligned} \quad (44)$$

are the additional functions associated with the new quantization and  $\theta$  is the angle between initial and final momentum of photon. With  $\alpha = e^2/4\pi$ , the differential cross section can then be written as

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{32m^4} \left( \frac{\omega'}{\omega} \right)^2 \left[ \frac{A}{\omega^2} + \frac{B}{\omega'^2} + \frac{C}{\omega\omega'} \right]. \quad (45)$$

In the point particle approximation, the function of  $\Psi(\vec{p})$  equals 1 and the above cross section is changed back to the traditional one

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{4m^2} \left( \frac{\omega'}{\omega} \right)^2 \left[ \frac{\omega'}{\omega} + \frac{\omega}{\omega'} + 4(\epsilon \cdot \epsilon')^2 - 2 \right]. \quad (46)$$

To test the solid quantization, it is interesting to measure cross section of high energy electron-photon Compton scattering because the function  $\Psi_e$  and  $\Psi_\gamma$  will have significant decrease at high momentum (energy). The smaller the particle, the larger the energy at which cross section has a clear difference from the traditional one.

Due to the inclusion of the size of the particle, the above fields ( $\phi(x)$  or  $\psi(x)$ ) as well as the propagators are not Lorentz covariant quantities. Our start point is that at each time  $t$ , each particle has a distribution on space. It is obviously non-relativistic though this physical picture is very clear and similar to many phenomenological models. It is also easy for us to apply this approach in numerical calculation. For example, in the effective field theory, finite regularization in which a  $\vec{k}^2$  dependent regulator  $u(\vec{k}^2)$  was introduced “by hand” was used to get rid of the divergence [32, 33]. We can use the above solid propagators for hadrons to investigate the meson loop contribution. Compared with finite range regularization, this approach automatically gives the “regulator” for each diagram. The obtained “regulator” is diagram dependent.

Now we give the relativistic version of the solid quantization. Different from the non-relativistic case, the field has a distribution on four dimensional space-time. For a scalar field  $\phi(x)$ , it can be written as

$$\phi(x) = \int \frac{d^4p}{(2\pi)^4} H(p^2) [\alpha_p e^{-ip \cdot x} + \alpha_p^\dagger e^{ip \cdot x}]. \quad (47)$$

The operators  $\alpha_p$  and  $\alpha_p^\dagger$  have the following commutation relations:

$$\begin{aligned} [\alpha_p, \alpha_q] &= [\alpha_p^\dagger, \alpha_q^\dagger] = 0, \\ [\alpha_p, \alpha_q^\dagger] &= (2\pi)^4 \delta^{(4)}(p - q). \end{aligned} \quad (48)$$

The commutation relations of scalar field and its conjugate are

$$\begin{aligned} [\phi(\vec{x}, t), \pi(\vec{y}, t)] &= \int \frac{d^4p}{(2\pi)^4} H^2(p^2) i p_0 (e^{i\vec{p} \cdot \vec{x}} + e^{-i\vec{p} \cdot \vec{x}}) \\ &= \int \frac{d^3p}{(2\pi)^3} \frac{i \Psi(\vec{p})}{2} (e^{i\vec{p} \cdot \vec{x}} + e^{-i\vec{p} \cdot \vec{x}}) \\ &\equiv i \Phi(\vec{x} - \vec{y}), \end{aligned} \quad (49)$$

where

$$\Psi(\vec{p}) = \int \frac{dp_0}{\pi} H^2(p^2) p_0. \quad (50)$$

For point particle with mass  $m$ ,  $\Psi(\vec{p}) = 1$  and  $H^2(p^2) = 2\pi\delta(p^2 - m^2)$ . We should mention that  $H(p^2)$  is proportional to  $\delta^{1/2}(p^2 - m^2)$  instead of  $\delta(p^2 - m^2)$ . This is because the field is expanded in terms of  $\alpha_p$  and  $\alpha_p^\dagger$  instead of  $a_p$  and  $a_p^\dagger$ .

For simplicity, we rewrite the scalar field as

$$\begin{aligned} \phi(x) &= \int \frac{d^4p}{(2\pi)^4} dM^2 H(M^2) \delta(p^2 - M^2) \\ &\quad [\alpha_p e^{-ip \cdot x} + \alpha_p^\dagger e^{ip \cdot x}] \\ &= \int \frac{d^3p}{(2\pi)^4 2\omega_M} dM^2 H(M^2) \\ &\quad [\alpha_{\vec{p}, \omega_M} e^{i\vec{p} \cdot \vec{x} - i\omega_M t} + \alpha_{\vec{p}, \omega_M}^\dagger e^{-i\vec{p} \cdot \vec{x} + i\omega_M t}], \end{aligned} \quad (51)$$

where  $\omega_M = \sqrt{\vec{p}^2 + M^2}$ .

We can get the propagator of scalar field as

$$\begin{aligned} \Delta_F(x' - x) &= \int \frac{d^3k}{4(2\pi)^4 \omega_{M'} \omega_M} dM^2 dM'^2 H(M^2) H(M'^2) \\ &\quad \delta(\omega_{M'} - \omega_M) [\theta(t' - t) e^{ik \cdot (x' - x)} + \theta(t - t') e^{ik \cdot (x - x')}], \end{aligned} \quad (52)$$

where  $\delta(\omega_{M'} - \omega_M) = 2\omega_M \delta(M'^2 - M^2)$ . With the definition of  $\theta$  function, the propagator can be written as

$$\Delta_F(x' - x) = \int \frac{d^4k}{(2\pi)^4} \frac{dM^2}{2\pi} \frac{i H^2(M^2)}{k^2 - M^2 + i\epsilon} e^{-ik \cdot (x' - x)}. \quad (53)$$

Again, if  $H^2(M^2) = 2\pi\delta(M^2 - m^2)$ , the propagator is the same as that for point particle with mass  $m$ . If  $H^2(M^2)$  is chosen to be  $2\pi[\delta(M^2 - m^2) - \delta(M^2 - \Lambda^2)]$ , one can get Pauli-Villars regularization.

In the relativistic case, the Lagrangian density can be written in the same way as Eq. (26) except the integral



is on four dimensional space-time because both time and space are non-local, i.e.

$$\mathcal{L}(x) = \int d^4a \bar{\psi}(x + \frac{a}{2}) e^{iI(x+a/2, x)} \gamma^\mu i D_\mu e^{-iI(x-a/2, x)} \psi(x - \frac{a}{2}) F(a), \quad (54)$$

where  $D_\mu = \partial_\mu - ig \int d^4b A_\mu(x+b) \frac{\Phi(a)\Phi_g(b)}{F(a)}$  and  $I(y, x) = g \int_x^y dz_\mu \int d^4b A^\mu(z+b) \frac{\Phi(a)\Phi_g(b)}{F(a)}$ . In the relativistic case, the function  $\Phi(a)$  or  $\Phi_g(b)$  is defined to be the Fourier transformation of  $1/\tilde{F}(p^2)$ , i.e.

$$\Phi(a) = \int \frac{d^4p}{(2\pi)^4} \frac{1}{\tilde{F}(p^2)} e^{ip \cdot a}, \quad (55)$$

where  $\tilde{F}(p^2)$  is the Fourier transformation of function  $F(a)$ . Similar as in the non-relativistic case, by comparing with the propagators obtained in solid quantization and path integral methods, one can get the relationship between  $H(p^2)$  and  $\tilde{F}(p^2)$ :

$$\int \frac{dM^2}{2\pi} \frac{H^2(M^2)}{p^2 - M^2} = \frac{1}{\tilde{F}(p^2)(p^2 - m^2)}. \quad (56)$$

Again, one can see that, to be consistent with the solid quantization, the Lagrangian is non-local in both free and interaction parts.

Due to the non-local property, the conservation laws are modified accordingly [34]. The currents or charges are not conserved at any space-time point. But the integral of them are conserved. For example, in the non-local case, there exists no unitary time evolution operator  $U(t_1, t_2)$  for given  $t_1$  and  $t_2$ . But there exists a unitary time evolution operator  $U(-\infty, \infty) \equiv T \exp\{i \int_{-\infty}^{\infty} d^4x \mathcal{L}_{int}(x)\}$ . This can be easily understood since a fermion, an anti-fermion and a gauge field can be annihilated/created at different time. The possibility of state is not conserved at a fixed time. But the integral of the possibility over the time is conserved. In other words, the time evolution operator  $U(-\infty, \infty)$  is unitary.

Though the relativistic version of the solid quantization is Lorentz invariant, the causality condition is different from the traditional quantum field theory. For example for scalar field in local case, the equal-time commutator  $[\phi(\vec{x}), \pi(\vec{y})]$  equal zero if  $x$  and  $y$  are spacelike separated, i.e.  $(x_0 - y_0)^2 - (\vec{x} - \vec{y})^2 < 0$ . In non-local case, there exists some possibility of non-zero commutator  $[\phi(\vec{x}), \pi(\vec{y})]$  for  $-(\vec{x} - \vec{y})^2 < 0$ . The non-zero commutator is because of the space-time distribution of the non-point fields. Therefore, the classic causality condition turns into a quantum (possibility) condition. Approximately, one may think the two non-point fields are spacelike separated if  $(x_0 - y_0)^2 - (\vec{x} - \vec{y})^2 < -\tilde{a}^2$ , where  $\tilde{a}$  is the size of the field.

In summary, we have proposed a new quantization - solid quantization for non-point fields. The divergence in

the loop integrals for point particles needs to be taken care of with the regularization method. This solid quantization condition is very natural and based on the idea that a physical particle is not a mathematic point one. The function in the commutation relations is another fundamental properties of the particle as well as mass, spin, width, etc. The divergence of loop integration could be systematically avoided from the beginning. Both non-relativistic and relativistic version of this solid quantization are given.

For the dimensional regularization, one has to use infinite Lagrangian or bare Lagrangian to get finite physical results. This method provide an interesting approach which is quite different from traditional quantum field theory. In this paper, we did not specify the function of  $\Phi(\vec{x})$ ,  $\Psi(\vec{p})$  or  $H(p^2)$ . To get more information about the function of the particle, it is important to do further numerical calculations to compare with the experiments and traditional results.

### Acknowledgements

The author would like to thank Prof. Y. B. Dong for helpful discussions.

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